STEADY-STATE MOTION OF RADIATION-HEATED VAPORS OF A MATERIAL IN THE PRESENCE OF LATERAL SPREADING FLOW

(STATSIONARNYI REZHIM DVIZHENIIA NAGREVAEMYKH IZLUCHENIEM PAROV VESHCHESTVA PRI NALICHII BOKOVOGO RASTEKANIIA)

PMM Vol. 31, No. 2, 1967, pp. 300-319

I. V. NEMCHINOV
(Moscow)

(Received September 24, 1966)

Radiation falling on the surface of a solid body vaporizes its material. A radiation-heated vapor jet is formed. With sufficiently prolonged heating the vapor propagates to distances comparable to the dimensions of the body, whereupon the jet begins to spread laterally. The vapor density diminishes more rapidly than in the case of planar motion, the transparency of the vapor increases, the radiation penetrates into the deeper layers of the material, and vaporizes them. A quasi-steady-state efflux of vapor into the vacuum begins.

The work performed by the expansion forces with increasing jet area near to the critical cross section (where the flow rate is equal to the speed of sound), is, for a certain ratio between the free path of the radiation and the cross section radius, compensated by heating due to the absorption of radiation. This makes possible the continuous acceleration of the gas and a transition from subsonic to hypersonic motion. Moreover, the "burnup" rate can be chosen on the basis of the aforementioned relationship between the radiation free path and radius of the solid (i. e. their approximate equality).

We shall consider the problem of motion in the case of spherical (or cylindrical) symmetry, with the rays also directed solely along the radii. In contrast to the cases of adiabatic motion or plane motion with heating, it is possible to have here steady efflux into the vacuum of gas from the core of the initially cold and slowly moving material, i. e. from the surface at which vaporization occurs (in the limiting case this is an infinitely dense, absolutely cold and stationary gas) with continuous acceleration and passage through the speed of sound.

The dependences of the mass burnup rate, pressure on the surface of the solid, and maximum gas temperature on the total incident radiation flux and sphere radius are considered for the case where the radiation absorption factor is constant or is a power function of the temperature and density.

The quasi-steady-state motion of a radiation-heated gas in a long, thin channel of constant cross section is also considered. On emerging from the channel the gas expands rapidly and becomes transparent. The burnup rate is determined by the length of the channel. The solutions considered show that lateral spreading flow can result in the establishment of a steady burnup proces and that the size of the body affects the burnup rate. These solutions reflect certain salient aspects of more complex motions involving lateral spreading flow

On reaching the surface of a solid, a powerful radiation beam vaporizes its surface material. Part of the radiation continues to be absorbed in the vapor, while the rest penetrates the vapor, falls on the surface of the solid, and continues to vaporize new layers. If the target body is placed in a medium characterized by sufficiently low pressure and density (in the limiting case, in a vacuum), the vapor expands rapidly away from its surface and forms a vapor stream. The vapor motion near the surface of the solid can be considered planar. In the case where the mass absorption coefficient $\varkappa$ is constant, the radiation is trapped in a practically constant mass on the order of $1/\varkappa$. After this mass has evaporated, there is no further vaporization, and the radiation energy is released in the constantly expanding vapor whose temperature increases [1]. With a variable absorption coefficient which diminishes with increasing temperature and decreasing density, the radiation can heat an ever larger mass; if, on the other hand, the motion is one-dimensional (planar), it cannot be steady, and the rate of burnup of the solid gradually diminishes, while the average vapor temperature rises slowly [2].

In studying the motion of material vapor heated by the radiation it absorbs one often encounters the problem of the effect of nonunidimensional flow. Thus, with radiation acting on the surface of a solid, the beam can be focused so strongly that its cross section and the cross section of the jet of escaping radiation-heated material (often called the "flare") become comparable to or even smaller than the length of this jet, i. e. the pressure gradients in the principal and lateral directions approach each other and the vapor motion becomes essentially two- or three-dimensional. This produces a drop in the gas density and an increase in the transparency of those layers of material in which lateral spreading flow occurs; these changes are more rapid than in the one-dimensional planar case. The mass and optical thickness of the layers of material in the beam path are smaller than in the one-dimensional case which does not involve lateral flow. Even when the mass absorption coefficient is constant, the radiation penetrates into the deeper layers of the material, which are also heated and begin to move. As a result, a heating and vaporization "wave" begins to move into the interior of the material. The rate of burnup of the material is then related to the size of the body or the radius of curvature of its surface, or, alternatively, to the size of the irradiated spot on its surface (if the diameter of the spot is smaller than the dimensions of the body or the radius of curvature of its surface), and finally, to the depth of the resulting "funnel" or "crater".

One can arbitrarily isolate the domain of two-dimensional motion characterized by lateral spreading flow and the domain of one-dimensional motion (at the surface of the body in the case of free spread of the jet or in the interior of the channel formed as a result of irradiation). Due to the rapid drop in density in it, the optical thickness of the divergent portion of the jet can be constant, so that a constant amount of radiation is applied to the domain of one-dimensional (planar) motion. This can result in steady motion and heating of the vapor and in steady burnup of the surface material.

If the characteristic vapor density is sufficiently low relative to the density of the solid material, the rate of vapor motion is considerably higher than the rate of progression of the boundary beyond which no material is being vaporized. For a certain amount of time the progression of this boundary and the change in the radius of curvature of the body relative to the initial radius can be neglected; i. e. the vapor can be assumed to be coming from the same surface. Establishment of steady motion of the radiation-heated vapor is, of course, possible only with a sufficient duration τ of the irradiation process. The duration τ must be larger than the time interval spanning the non-steady-state heating and expansion of the vapor to distances at which the jet length exceeds a certain value, which is in any case larger than the radius of curvature. The size of the body changes as it burns up. With slow variation of the radius of curvature of the body, the motion is quasi-steady-state, corresponding to the instantaneous value of this radius at each instant.

Computation of the gas motion in the case of a complex streamline configuration and with allowance for the absorption of radiation is, by virtue of the two-dimensional character of the problem, quite difficult. This also applies to the determination of the burnup rate and pressure at various points of the vaporization surface. We shall consider a problem for which such computations can be carried out fairly simply, and for which we can determine the basic features of the motion and heating processes with lateral flow. These features remain applicable in more complicated instances.

Let the radiation fall strictly radially part of the radiation penetrating to the surface of a solid sphere (or cylinder) of radius r_0 and vaporizing the material of this surface, and the rest of the radiation being absorbed in the vapor to increase their total energy per unit mass (i. e. heating or expanding the vapor, and sometimes increasing simultaneously its inner and kinetic energy). Let the motion be confined to the radial direction. This is possible if the radiation falls on the sphere (or cylinder) from all sides or if the body lies "at the bottom" of a cone with straight generatrices and rigid walls which limit the size of the gas jet; here the radiation fills the entire cone, and heat conduction to the walls (and throughout the gas) can be neglected.

As we know [3], with planar steady-state motion of a heated gas in a tube of constant cross section there can be no transgression of the speed of sound. The only possibility is subsonic motion with acceleration due to heating. Application of heat to a hypersonically moving gas decelerates it. We also know [3 and 4] that spherically or cylindrically symmetrical adiabatic motion of a gas can be either subsonic or hypersonic, since the "sonic" surface which in this case bounds the source core is the site of infinite accelerations of the gas. With subsonic radial motion in an expanding tube the gas is decelerated; in the case of hypersonic motion it is accelerated (Laval nozzles in which gases are accelerated to hypersonic velocities have a narrow portion at one end).

In the case of the heated gas flow to be considered in the present paper, the area of the surface cross section through which the gas flows increases continuously, and the gas experiences continuous heating. But if both factors are operating simultaneously, i. e. if heating predominates near the surface of the sphere, while the dominant forces at the periphery are those of expansion, and if near some critical cross section the work of the expansion forces is, in a manner of speaking, compensated by radiation heating (the speed of the gas is equal to the speed of sound in this cross section), it is possible to have continuous acceleration of the gas up to hypersonic velocities. The heating intensity is related to the free path of the radiation, which is variable (this happens, for example, when the density varies while the mass absorption coefficient remains constant); the work of the expansion forces depends on the critical cross section, so that the determination of all the parameters in the critical cross section, i. e. the "selection of the burnup rate, can be carried out on the basis of a specific relationship between the radius of this cross section and the radiation free path in it.

1. The system of equations of steady-state motion and heating of vapor in the case of radial symmetry can be written as

$$\zeta(\nu)\,\rho u r^{\nu-1} = M^{\cdot}, \qquad dp + \rho u\,du = 0$$

$$M^{\cdot}(h + {}^1/_2\, u^2 + Q) + F = M^{\cdot}(h_s + {}^1/_2\, u^2{}_s + Q) + F_s \qquad (1.1)$$

$$\frac{dF}{dr} = F\varkappa\rho, \quad h = \frac{p}{\rho}\frac{\gamma}{\gamma - 1}$$

Here p is the pressure, ρ the density, u the velocity, h the enthalpy of the material, and $\varkappa$ the mass absorption coefficient; these parameters refer to a cross section with the radius r. F is the total radiation flux through the entire surface, M the (constant) mass consumption rate, Q the heat of vaporization of the material, and γ the adiabatic index, $\nu = 1, 2, 3$ in the plane, cylindrical, and spherical cases; the corresponding values of the coefficients $\zeta(\nu)$ are $1, 2\pi$, and 4π, respectively. The boundary conditions for the differential equations of Systems (1.1) are

$$p \to 0, \qquad F \to F_\infty, \qquad r \to \infty$$

The dependence of the absorption coefficient $\varkappa$ on the thermodynamic parameters is considered known, i.e. the function $\varkappa\ (h,\ \rho)$ or $\varkappa\ (p,\ \rho)$ is given. We note that the radiation free path is $l = 1\ /\ \varkappa\rho$.

The reaction of the departing vapor increases the pressure on the surface of the evaporating body. However this pressure is usually small as compared with the bulk compression modulus of the body, and the change in the density of the material in front of the heating and vaporization wave under the action of this pressure is negligible as compared with the normal density ρ_0. The radiation free path l_0 in the solid is often extremely small, and the thickness of the zone in which heating to the boiling point T_k at the pressure p_0, produced in the solid occurs is on the order of l_0, and is negligible as compared with the radius r_0 of the body. Between the zone where vaporization has terminated, i.e. where there are no liquid droplets, and the zone where it has not begun, i.e. where there are no vapor bubbles (in the case of quasi-equilibrium vaporization) we have a transitional zone where the pressure p varies little in comparison with p_0 This zone is further characterized by still weaker variation of the equilibrium temperature of the vapor and liquid (i.e. the boiling point T_k), but by substantial variation of the vapor and liquid concentrations, as well as by marked variations in the average density ρ of the mixture and its enthalpy h as a result of the vaporization heat expenditure Q. We shall assume that the absorption factor of the vapor at $T \approx T_k$ and $p \approx p_0$ is sufficiently high. This enables us to neglect the width of the transitional (vaporization) zone.

Thus, if the indicated assumptions are fulfilled, the transitional zone and the zone where the material is heated to the boiling point can be replaced by a gap, assuming that $r_s = r_0$. The subscript s in Equations (1.1) and below denotes parameters at the outer boundary of the vaporization zone.

From the conservation laws at the gap we obtain (the subscript 0 denotes quantities behind the gap

$$p_s + \rho_s u_s{}^2 = p_0, \quad M^{\cdot} = \zeta\rho_0 u_0 r_0^{\nu-1} = \zeta\rho_s u_s r_s'^{\nu-1} \qquad (1.2)$$

$$M^{\cdot}(h_s + {}^1/_2\, u_s{}^2 + Q) + F_s = M^{\cdot}(h_0 + {}^1/_2\, u_0{}^2) + F_0$$

Here, u_0 is the velocity of the material entering the gap. The density of the material before it enters the gap is normal density ρ_0 of the solid body. The pressure p_0 behind the gap is higher than the pressure p_s ahead of it, and both of these quantities are unknown, since the gap moves through the material at a subsonic velocity relative to the material behind and ahead of it.

This implies that $h_s = h(T_k, p_s)$, where T_k is the boiling point; the function $T_k(p_s)$, and therefore $h_s(p_s)$, must be given. At pressures substantially lower than the critical pressure the vapor can be assumed to be an ideal gas, i.e. the last Eq. in (1.1) is also valid for quantities with the subscript s. The quantity h_s varies relatively little even with substantial variation of the pressure p_s (for $p < p_T$, where p_T is the critical pressure). Since the density ρ_s of the gas emerging from the vaporization zone at low pressures p_s is considerably lower then the density ρ_0 of the solid, its velocity u_0 is small as compared with the velocity u_s of the gas ahead of the gap (following vaporization); the kinetic energy $u_0^2/2$ of the unvaporized material entering the gap is negligible as compared with the kinetic energy $u_s^2/2$ of the vapor and its enthalpy h_s ; u_0 and $u_0^2/2$ can be therefore neglected.

If the initial temparature T_0 of the material is low as compared with the boiling point T_k it can be also neglected. Thus, we set $h_0 = 0$, $u_0 = 0$, $F_0 = 0$, $\rho_0 = \infty$.

If the maximum temperature T_{max} of the vapor attained during its heating is considerably higher than the boiling point T_k further if the vapor density in the zone where the maximum temperature has been attained is considerably higher than the density ρ_s and especially ρ_0, then we can set $T_k = 0$ and $h_s = 0$, and since the pressure is finite, it follows the $\rho_s = \infty$ (or $v_s = 1/\rho_s = 0$), and, consequently, $u_s = 0$ ($M^{\cdot}$ remains finite) and $p_s = p_0$. With sufficient heating of the vapor one can also neglect the heat of vaporization Q; then, of course, $F_s = 0$. With strong heating of the vapor, assumptions about the quasi-equilibrium character of the process of transition from the solid to the gaseous state likewise become immaterial and there is no longer any need to introduce the gap. Thus, the vaporization surface gives off a cold and slowly moving gas. In some cases we shall assume that this surface gives off (from the core of the source) an absolutely cold and stationary, infinitely dense gas at finite pressure.

Thus, four conditions at the gap (the right-hand side of the third Eq. of (1.1) is equal to zero) relate six unknowns: $\rho_s, p_s, F_s, u_s, p_0$ and $M^{\cdot}$. By specifying two of them (e.g. the radiation flux F_s applied to the vaporization surface and the pressure p_s at the surface), we can determine all of the quantities, including p_0 and $M^{\cdot}$, and begin to integrate the two differential Eqs. of the system (1.1) with allowance for its algebraic relations from the point $r = r_s = r_0$. On integrating to $r = \infty$, we obtain the value of the total incident radiation flux F_∞ corresponding to the given parameters at the vaporization wave, i.e. to its specified velocity and pressure. First, however, it is necessary that the flux F_∞ be finite; second, the condition $p_\infty = 0$ must be fulfilled. This is possible only if these quantities are specified not arbitrarily, but in a specific relationship to each other which, as will be shown below, is found by analyzing the system of equations itself.

As already noted and as will be demonstrated below, the condition which enables us to determine all the parameters uniquely is the condition of passage through the speed of sound in some critical cross section ($u^2 = \gamma p / \rho$). We denote all parameters in this cross section by the subscript $*$.

Making use of the condition of passage through the speed of sound, we find from Eqs. (1.2) that

$$\zeta(\nu)\,\rho_* u_* r_*^{\nu-1} = M^{\cdot}, \qquad (\gamma-1)\,h_* = \gamma p_* / \rho_* = u_*^2, \qquad M^{\cdot}\,[{}^1/_2(\gamma+1)\,h_* + Q] = -F_* \tag{1.3}$$

We recall that the radiation flux is directed toward the body, so that $F_* < 0$. In analyzing the equations it is more convenient to consider h_*, F_*, and r_* as the given quantities rather than h_s, F_∞ and r_0.

Let us refer all of the quantities h, p, ρ, u, F, r, $\varkappa$ and l to the corresponding values in the critical cross section, denoting these new dimensionless quantities by the same letters without subscripts. Eqs. (1.1) then become (1.4)

$$\rho u r^{\nu-1} = 1, \qquad dp + \gamma\rho u\, du = 0, \qquad h + {}^1/_2 u^2 (\gamma - 1) + \chi = F\left({}^1/_2(\gamma + 1) + \chi\right)$$

$$\frac{dF}{dr} = \frac{F\varkappa\rho}{\lambda} = \frac{F}{\lambda l}, \qquad h = \frac{p}{\rho}$$

(we recall that the right-hand side of the third Eq. of this system is equal to zero).

The dimensionless radiation flux $F > 0$, and the dimensionless criteria χ and λ have the following values:

$$\chi = Q / h_*, \quad \lambda = l_* / r_* \tag{1.5}$$

Relations (1.3) become

$$M^{\cdot} = \frac{(-F_*)}{h_*\left({}^1/_2(\gamma + 1) + \chi\right)}, \qquad \rho_* = \frac{M^{\cdot}}{\zeta(\gamma - 1)^{1/2} h_*^{1/2} r_*^{\nu-1}} \tag{1.6}$$

Thus, if F_*, h_*, and r_* are given, we can determine ρ_*, p_*, $\varkappa_*$ and $l_* = 1 / \varkappa_*\rho_*$, and therefore the parameters λ and χ. For the critical combination we have

$$h = p = \rho = u = F = r = 1 \tag{1.7}$$

Integrating the system in the domain $r > 1$, we find the dimensionless quantity $F(r \to \infty)$, i.e. the dimensional value of the total incident radiation flux F_∞; integrating in the domain $r < 1$, we find the relationship between $h(r)$ and $p(r)$ and can determine the ratio R of the body radius r_0 to the specified radius r_* of the critical cross section as a function of the ratio $h(R)$ of the vapor enthalpy h_s to the arbitrarily specified quantity h_*. Thus, the true size of the critical cross section and the true value of the gas enthalpy in it remain unknown. But the point where conditions (1.7) are fulfilled is a singular point of system (1.4). Passage through this point along the single integral curve passing through it requires fulfillment of an additional condition for λ; in other words, a specific relationship must exist between the radiation free path l_* in the critical cross section and the radius r_* of this section, and therefore between p_* and F_*. The need for this condition is apparent from the qualitive arguments set forth above.

2. We shall now determine the quantity λ on the basis of the condition of continuous acceleration of the gas. Introducing

$$S = r^{\nu-1}, \qquad g = u^2, \qquad \omega = (\nu - 2) / (\nu - 1) \tag{2.1}$$

and differentiating the algebraic relations of system (1.4), we obtain

$$\frac{dF}{F} = \frac{\varkappa}{\lambda(\nu - 1) g^{1/2} S^\omega} \frac{dS}{S}, \qquad \frac{d\rho}{\rho} + \frac{dg}{2g} + \frac{dS}{S} = 0, \qquad \frac{dp}{p} + \frac{\gamma g}{2h}\frac{dg}{g} = 0$$

$$\frac{dh}{h} + \frac{(\gamma - 1) g}{2h}\frac{dg}{g} = \frac{F}{h}\left(\frac{\gamma + 1}{2} + \chi\right)\frac{dF}{F}, \qquad \frac{dh}{h} + \frac{d\rho}{\rho} - \frac{dp}{p} = 0 \tag{2.2}$$

After the appropriate eliminations this system yields

$$\frac{dg}{g}\frac{(h - g)}{h} = 2\frac{dS}{S}\left(\frac{F\varkappa\left[{}^1/_2(\gamma + 1) + \chi\right]}{\lambda(\nu - 1) h g^{1/2} S^\omega} - 1\right) \tag{2.3}$$

Expression (2. 3) clearly implies the following. In order for the gas to accelerate continuously in passing through the speed of sound ($h=g=u^2$) it is necessary that Eq.

$$\frac{l_*}{r_*}=\lambda=\frac{(\gamma+1)+2\chi}{2(\nu-1)} \tag{2.4}$$

be fulfilled in the critical cross section ($S=F=1, \varkappa=1, h=g=1$).

Otherwise, with $h=g$, in accordance with (2. 4) the gas acceleration $(dg/dS)=\infty$ and changes sign.

In order for the gas to continue to accelerate it is necessary that the change of sign $(h-g)$ be accompanied by a change in the sign of either the quantity dS or the expression in parentheses in the right-hand side of (2. 3). This occurs if (2. 4) is fulfilled. The quantity dS cannot change sign, since we have accepted the condition of continuous increase of the jet cross section with distance from the body (a nozzle with straight walls), i. e. dS/dr does not change sign. The necessity for acceleration of the gas follows from the fact that, by hypothesis, the efflux is into a vacuum, so that the motion is hypersonic for $p\to 0$.

This is clear in the case of adiabatic motion. But the major effect of heating lies solely in the fact that the motion at large distances from the critical cross section is isothermal and the quantity h is finite (h cannot increase continuously with a finite total incident radiation flux F by virtue of the energy balance equation of sytem (1. 1) or (1. 4)). Thus, the kinetic energy g and the maximum velocity of motion u_m must also be finite.

Let us write the equation of moments of system (1. 4) for the case of isothermal motion,

$$\frac{dp}{p}+\frac{u\,du}{h}=0, \qquad p=P\exp\left(-u^2/2h\right) \tag{2.5}$$

Here P is an integration constant. Clearly, the condition $p\to 0$ implies that $u/\sqrt{h}\to\infty$, i. e. the motion becomes hypersonic. We note that isothermal motion is a limiting case which, strictly speaking, is not realized, since a finite h implies a finite u.

When condition (2. 4) is fulfilled, Eqs. (2. 2) and (2. 3) become (for $\chi=0$, i. e. for $h_* \gg Q$)

$$\frac{dF}{F}=\frac{2}{(\gamma+1)}\frac{\varkappa}{g^{1/2}}\frac{dS}{S^{\omega}}, \qquad h={}^1/_2\left[F(\gamma+1)-(\gamma-1)g\right] \tag{2.6}$$

$$^1/_2(\gamma+1)(F-g)S\,dg=2g^{1/2}(F\varkappa S^{-\omega}-hg^{1/2})dS$$

As is evident, the critical point becomes a singular point of system (2. 6), and it is then possible to avoid infinite accelerations in passing through the speed of sound and changes in the sign of (dg/dS) provided there exist integral curves passing through this point. As will be shown below, such a curve does, in fact, exist, and is unique.

Integrating (2. 6), we can determine the dependences of F and g on S, and hence on h, ρ, and p.

In order to find these dependences we must specify the relationship between the absorption coefficient and the thermodynamic parameter h and ρ or p and ρ. We shall assume that this relationship is exponential,

$$\varkappa=Kh^{-\alpha}\rho^{\beta} \tag{2.7}$$

With a constant absorption coefficient ($\varkappa$ = const) $\alpha = \beta = 0$. When radiation is absorbed by a completely ionized gas (free-free absorption) [5 and 6], $\alpha = 3/2$ and $\beta = 1$, and the coefficient $K \sim 1/(h\nu)^2$, where $h\nu$ is the energy of the radiation quanta. In a multiple ionization zone [6] the function $\varkappa(h, \rho)$ can also be expressed approximately in exponential form.

3. It is generally necessary to find the maximum heating, h_{max}, for given values of F_∞ and r_0. Neglecting absorption in the hypersonic portion of the nozzle, we have $h_{max} \approx h_*$ and $F_\infty \approx F_*$. By (2.4), for $\nu = 3$ and $\chi = 0$ the parameter $\lambda = (\gamma + 1)/4$, i.e. the free path of radiation in the critical cross section ℓ_* is 1.5 to 2 times smaller than the radius r_* of this cross section. Hence, the layer where the gas is heated from $h = h_s \ll h^*$ to $h = h_*$ is on the order of r_0, and we can assume that r_* is on the order of the body size r_0. Let us evaluate the accuracy of these assumptions by computing the optical thickness of the adiabetic jet in which the gas moves at hypersonic velocity without heating and by determining the width of the heating zone in a gas moving with subsonic velocity in a one-dimensional (planar) configuration in such a way that the Jouget rule is fulfilled at the boundary of this zone.

We begin by writing out the relationship for the parameters in the adiabatically expanding gas,

$$h = \left[1 - \frac{(\gamma - 1)\, u^2}{\gamma + 1}\right] \frac{\gamma + 1}{2}, \qquad p = \rho^\gamma = h^{\gamma/(\gamma - 1)}, \qquad S = r^{\nu - 1} = \frac{1}{u\rho} \tag{3.1}$$

Thus, all of the quantities are expressible in terms of the single parameter u, i.e. in terms of the ratio of the gas velocity to the velocity of the gas and the speed of sound in the critical cross section (as before, we shall make use of dimensionless quantities).

In a gas whose parameters vary in accordance with (3.1) and whose $\varkappa$ obeys law (2.7), the optical thickness τ_2 is given by the Expression

$$\varkappa = h^{-\alpha} \rho^\beta = \rho^b, \qquad b = \beta - (\gamma - 1)\,\alpha \tag{3.2}$$

$$\lambda \tau_2 = \int_1^\infty \varkappa \rho\, dr = \int_1^\infty \rho^{b+1}\, dr \tag{3.3}$$

We must note that for $\gamma = 5/3$, $\alpha = 3/2$, and $\beta = 1$ the absorption coefficient is constant provided the gas parameters vary adiabatically, but $\ell \to \infty$ as expansion progresses. With $\gamma < 5/3$ the index $b > 0$, i.e. the absorption coefficient $\varkappa \to 0$ as the density decreases. For example, $b = 2/5$ for $\gamma = 7/5$.

Making use of relation (3.1), we obtain

$$r = (u\rho)^{-1/(\nu - 1)}, \qquad dr = \frac{(u\rho)^{-1/(\nu - 1)}}{\nu - 1} \frac{1 - u^2}{1 - u^2(\gamma - 1)/(\gamma + 1)} \frac{du}{u} \tag{3.4}$$

Hence, Expression (3.3) becomes

$$\tau_2 = \left(\frac{\gamma + 1}{2}\right)^a \int_1^{u_m} \left(1 - \frac{\gamma - 1}{\gamma + 1} u^2\right)^c u^{-\nu/(\nu - 1)} (u^2 - 1)\, du \tag{3.5}$$

$$c = \frac{b + (\nu - 2)/(\nu - 1)}{(\gamma - 1)} - 1, \qquad u_m = \frac{u_{max}^{(a)}}{u_*} \tag{3.6}$$

$$a(\gamma - 1) = b - (\gamma - 1)$$

Here u_m is the ratio of the maximum gas velocity with adiabatic motion $u_{max}^{(a)}$ to the velocity in the critical cross section. In accordance with (3. 1), $u_m^2 = (\gamma+1)/(\gamma-1)$. In deriving (3. 5) we made use of Expression (2. 4) for $\chi = 0$.

For $\nu = 3$ and $\gamma = 5/3$ we find that $c = -1/4$; for $\gamma = 7/5$ it turns out that $c = 5/4$. The corresponding values of τ_2 in accordance with (3. 5) are 0. 70 and 0. 36.

Considering the variation of the parameters at a sufficient distance from the critical cross section, i. e. where $u \approx u_m$, we arrive at the approximate expressions

$$\varkappa = u_m^{-b} r^{-(\nu-1)b}, \qquad \rho = u_m^{-1} r^{-(\nu-1)} \tag{3.7}$$

Using this expression throughout the domain $1 \leqslant r \leqslant \infty$, we obtain the following approximate value for τ_2:

$$\lambda\tau_2 = u_m^{-(b+1)} [(\nu-1)(b+1) - 1]^{-1} \tag{3.8}$$

Since (3. 7) is valid for $r \to \infty$, it is easy to determine from (3. 8) whether the optical thickness of a jet (along its axis) of adiabatically expanding gas is a finite quantity.

Integrals (3. 5) and (3. 8) diverge for $b \leqslant (2-\nu)/(\nu-1)$. In the cylindrical case the optical thickness is definite if $b \leqslant 0$, in the spherical case it is infinite if $b \leqslant -1/2$. Thus, in the cylindrical case (motion in a "wedge-shaped slot") with infinitely long walls or in the case of motion from an infinitely long cylinder irradiated from all sides perpendicularly to its surface, the optical thickness is infinite if $\varkappa = \text{const}$ ($b = 0$) (finite in the spherical case). This can indicate either that a steady state is impossible, or that adiabatic motion is impossible, so that heating cannot be neglected if the expansion of the jet area happens to be large.

With isothermal motion $b = \beta$, so that in the cylindrical case the optical thickness is infinite and steady-state motion is impossible for $\beta \geqslant 0$, in the spherical case it is impossible only for $\beta \geqslant -1/2$.

Since in the spherical case for $\varkappa = \text{const}$ or for $\alpha = 3/2$ and $\beta = 1$ for any γ we have $b > -1/2$, the optical thickness τ_2 is finite, and, since τ_2 is on the order of unity, absorption by the hypersonic portion of the jet is comparatively slight, while at some distance from the critical cross sections the parameters vary adiabatically.

4. Let us consider the planar motion of a heated gas ($\nu = 1$). From the first and second Eqs. of (1. 4) we have

$$\rho u = 1, \qquad p = 1 + \gamma - \gamma u \tag{4.1}$$

From relation (4. 1) we see that as $u \to 0$, i. e. in the case where the gas entering the heating wave is absolutely cold and infinitely dense, the pressure $p_s = p_0$ at the heating wave boundary exceeds the pressure at the critical point p_* by a factor of just $(\gamma + 1)$, i. e. the pressure p_* also characterize the pressure p_0 at the body surface.

The radiation transfer equation in the case of planar motion can be written as

$$\frac{dF}{dr} = \frac{F}{\lambda l} \tag{4.2}$$

With a function $\varkappa(h, \rho)$ described by relation (2. 7) we have

$$l = h^{\alpha} \rho^{-(\beta+1)} = p^{-(\beta+1)} h^{\alpha+\beta+1} \tag{4.3}$$

In accordance with the third and fourth Eqs. of (1. 4) we obtain

$$F = u\,(2 - u), \qquad h = u\,(1 + \gamma - \gamma u) \tag{4.4}$$

Hence, Eq. (4. 2) becomes

$$1 - r = \lambda \int_F^1 l F^{-1}\,dF = \lambda J\,(u) \tag{4.5}$$

$$J\,(u) = \int_u^1 2\,(1-u)\,(2-u)^{-1}\,u^{\alpha+\beta}\,(1+\gamma-\gamma u)^{\chi}\,du$$

The quantity $J(0)$, i. e. the integral in the right-hand side of (4. 5) with the integration limit $u=0$ is the equal to 0. 614 for $\alpha=\beta=0$, i. e. $R = 1 - 0.614\lambda$. For $\alpha = 3/2$, $\beta = 1$ for $\gamma = 5/3, 7/5$, and $6/5$ we obtain $J(0) = 0.188, 0.172$, and 0.160, respectively.

Before turning to the determination of the precise distribution of the quantities, we take note of the nonplanar character of the motion and make the following remark which appears to be of some practical interest. Heretofore we have assumed that the problem involves just one characteristic dimension, i. e. the size of the body or the size of the irradiated spot on its surface. In general, this is, of course, not so. The action of focused radiation on the surface of a body often produces a deep, narrow channel. The radius r_0 of the transverse cross section ceases to be a major parameter, and is replaced by the channel depth L. We assume that on emerging from the channel the gas expands rapidly, its density diminishing rapidly at distances (from the channel exit) on the order of $r_0 \ll L$, the two-dimensional portion of the jet is completely transparent. With slow changes in channel depth, the motion can be assumed quasi-steady-state. If the area of the transverse channel cross section varies little, then to describe the variation of parameters within it we must make use of relations (4. 1) to (4. 4). The conditions dictating the selection of burnup parameters are the Jouguet rules at the channel exit and the conditions stipulating a given thickness of the steadystate heating zone (i. e. it is made equal to the channel depth L). Condition (2. 4), i. e. the relationship between l_* and r_0 ceases to be fulfilled, and the quantity l_* is found from another condition

$$\lambda = l_* / L = 1 / J\,(0) \tag{4.6}$$

Thus, for $\alpha=\beta=0$ the radiation free path in the exit cross section of the channel is 1. 6 times larger than the channel length; for $\alpha = 3/2$ and $\beta = 1$ it is 5-6 times larger. All of the remaining parameters can be found from Eqs. (1. 3) where F_* amd M^* are interpreted as the mass burnup rate and the radiation flux through the whole of the channel cross section. For $\alpha = 3/2$ abd $\beta = 1$ we obtain (for $\chi = 0$)

$$h_* = (\gamma - 1)^{-2/9}\,[{}^1/_2\,(\gamma + 1)]^{-4/9}\,(\lambda K)^{-2/9}\,q_*^{4/9} L^{2/9} \tag{4.7}$$

Here q_* is the radiation flux per unit area of the exit cross section.

Thus, the temperature of the gas at the channel exit increases with the gradual increase in channel depth, and the density gradually diminishes. This has the effect of increasing the radiation free path and reducing the optical thickness of each of the channel segme-ments (if the material passing through the bottom of the channel is assumed to be an infinitely dense and absolutely cold gas, the total optical thickness of the channel is infinite).

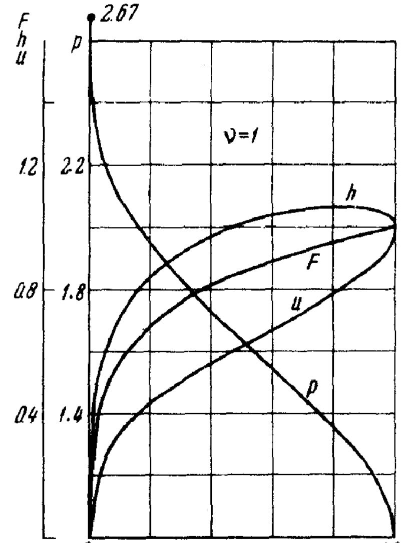

Fig. 1

Fig. 1 shows the distribution of gas parameters as a function of channel depth for $\varkappa$ = const.

As in the case of an expanding nozzle with straight walls, the establishment of a quasi-steady-state in a channel of constant cross section involves lateral spread of the jet accompanied by increasing transparency of the gas.

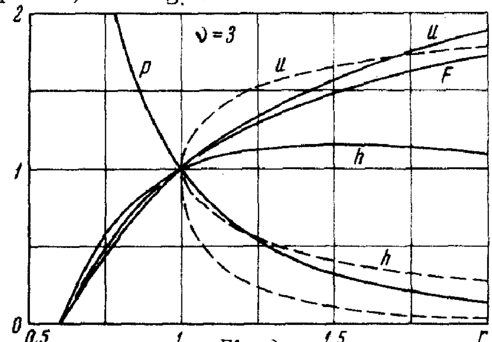

Fig. 2

Steady-state and nonsteady-state motion and heating of a gas in channels of variable cross section can be considered approximately with the aid of quasi-unidimensional motion Eqs. [3 to 7] adapted to take account of heating. This does not require that the cross section of the channel be given. With free lateral spread of the jet we may add an Eq. which describes approximately the variation of jet radius with changes in the pressure gradients in the transverse cross section. We have, of course, invariably assumed that the vapor is capable of absorbing a substantial portion of the radiation, so that the drop in the mass burnup rate and density in the critical cross section would result in a marked increase in the radiation free path and thus in the amount of applied heat, which would in turn increase the mass burnup and density. If, on the other hand, the vapor is perfectly transparent, absorption occurring only in the unvaporized or incompletely vaporized material, then, of course, neither the size of the surface spot nor the channel depth determine the burnup rate, and the motion of gas in the jet or in the channel is purely adiabatic.

A steady state can also arise when all heating ceases upon attainment of a certain temperature (e. g. through the dissociation of the absorbing molecules). The same thing can happen when the density drops below a certain limit or when there is a transformation to another phase state, provided these effects alter the radiation absorption mechanism markedly and produce a sharp drop in the absorption coefficient. With such motions the burnup rate is determined not only by the Jouguet rule, but by yet another physical parameter such as the transparency temperature. All quantities on such a plane heating and vaporization wave can be determined with the aid of relation (1. 3) for $\nu = 1$ by specifying one of the parameters, e. g. h_*,

$$M^{\cdot} = \frac{F_*}{H}, \qquad p_* = \frac{F_* (\gamma - 1)^{1/2} h_*^{1/2}}{H}, \qquad H = {}^1/_2 (\gamma + 1) h_* + Q \tag{4.8}$$

Here the parameters in the heating and vaporization wave are independent of the body size. The quantity H represents the effective "burnup" enthalpy; it is sometimes the case that $H \gg Q$ if $T_* \gg T_k$.

On the other hand, for $\nu = 2$ and $\nu = 3$, a steady state for the motions considered is established even if heating does not cease "for physical reasons".

5. We shall now construct precisely the parameter distributions for the spherically and cylindrically symmetrical motion of radiation-heated gas involving passage through the speed of sound. This will enable us to determine the relationship between the maximum heating h_{max}, the pressure p_0 at the surface of the body, and the body radius r_0 on the one hand, and the parameters h_*, p_*, and r_* at the critical cross section on the other. Taking account of its integrals (see (1.4)), we have already reduced system (2.2) to a system of two ordinary Eqs. (2.5). Expanding $S(g)$ and $F(g)$ in a series near the singular point $F=g=S=1$, we obtain

$$S-1={}^1/_2(\gamma+1)Z(g-1) \qquad F-1=Z(g-1) \qquad (5.1)$$

The slope Z of the integral curve $F(g)$ passing through the singular point is found by solving the quadratic Eq.

$$aZ^2+bZ-1=0 \qquad (5.2)$$

In the spherical case ($\nu=3$), when the dependence of the absorption coefficients on the thermodynamic parameters is exponential (2.7), the coefficients a and b are given by the Eqs.

$$a={}^1/_2(3\gamma-1)+2\alpha+\beta\gamma, \qquad b=3-\gamma-\beta-\alpha(\gamma-1) \qquad (5.3)$$

$$a={}^1/_2(3\gamma-1), \quad b=3-\gamma, \alpha=0, \quad \beta=0 \qquad (5.4)$$

$$a={}^5/_2(\gamma+1), \quad b={}^1/_2(7-5\gamma), \quad \alpha={}^3/_2, \beta=1$$

For $\alpha=0$, $\beta=0$ we have $Z=0.445$ for $\gamma=5/3$ and $Z=0.435$ for $\gamma=7/5$; for $\alpha=3/2$ and $\beta=1$ we obtain $Z=0.440$ for $\gamma=5/3$ and $Z=0.408$ for $\gamma=7/5$. Thus, the slope Z changes little with α, β, γ, i.e. the functions $F(g)$ also changes little.

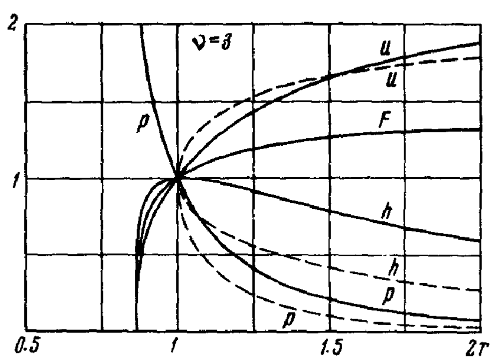

Fig. 3.

Proceeding from the singular point in accordance with Eqs. (5.1), we can integrate system (2.5) numerically. The results appear as the solid curves in Fig. 2 for $\alpha=0$ and $\beta=0$ and in Fig. 3 for $\alpha=3/2$ and $\beta=1$, as well as in the table (for $\nu=3$).

These data indicate that, in fact, for $\alpha=3/2$ and $\beta=1$ the parameter distribution can be approximately described by relations (3.1) in the hypersonic portion of the nozzle (broken curves in Figs. 2 and 3) and by relations (3.9) and (3.12) in the subsonic portion. Near the critical cross section, where the radiation is largely absorbed by the hypersonic portion of the jet, the motion is practically isothermal. Hence, the exponent α has little effect on the ratio of the flux F_* in the critical cross section to the incident

α	β	γ	$\frac{F_\infty}{F_*}$	$\frac{h_{max}}{h_*}$	$\frac{u_{max}(a)}{u_*}$	$\frac{u_\infty}{u_*}$	$\frac{p_0}{p_*}$	$\frac{r_0}{r_*}$
0	0	$^6/_5$	2.47	1.54	3.33	4.96	2.80	0.670
		$^7/_5$	2.42	1.29	2.45	3.78	3.20	0.648
		$^5/_3$	2.35	1.15	2.00	3.06	3.78	0.620
$^3/_2$	1	$^6/_5$	1.30	1.06	3.33	3.66	2.10	0.904
		$^7/_5$	1.35	1.02	2.45	2.83	2.34	0.891
		$^5/_3$	1.42	1.00	2.00	2.37	2.64	0.874

flux F^{∞}.

The maximum enthalpy is, in fact, close to h_*, and the pressure p_0 on the surface of the body is close to $(\gamma+1)p_*$; the radius r_0 of the body does not differ greatly from the radius r_* of the critical cross section. For $\alpha=\beta=0$ the disparity between the results of numerical integration from those obtained by approximation is more substantial.

6. Let us consider the cylindrical case ($\nu=2$). Although, as was shown above by means of estimates, steady-state motion is impossible in some cases because the optical thickness of the hypersonic portion of the jet is infinitely large with infinitely long "wedge-shaped slot" walls. However, if these walls are of finite length and the slot is narrow, so that the gas entering it expands rapidly and becomes transparent, or if the gas ceases to be heated (for physical reasons) starting at some transparency temperature, then such motion is possible. Let us analyze this case in greater detail. This will enable us to use a similar approach in analyzing the somewhat more complex spherical case.

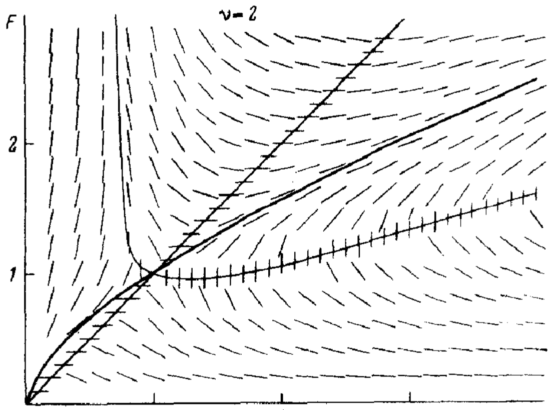

Fig. 4.

For $\varkappa=\text{const}$ system (2.5) can be transformed into a single Eq. for F and g,

$$F\,(F-g)\,dg = 2g\,[F - g^{1/2}\,{}^1\!/_2\,((\gamma+1)F-(\gamma-1)g)]\,dF \qquad (6.1)$$

Fig. 4 shows the field of integral curves of Eqs. (6.1). The line $dg/dF=0$ is described by Eq.

$$F=\varphi(g) = {}^1\!/_2\,(\gamma-1)g^{3/2}[{}^1\!/_2 g^{1/2}(\gamma+1)-1]^{-1} \qquad (6.2)$$

For large g this line is the straight line $F=g(\gamma-1)/(\gamma+1)$. However, this limiting curve is attained only for $g\to\infty$ and $F\to\infty$. On the line $F=g$ where passage through the speed of sound occurs, $dg/dF=\infty$ and dF changes sign; but this cannot happen, since the radiation is merely absorbed.

The integral curve $F(g)$ corresponding to a state of continuous acceleration of the gas and to passage through the speed of sound passes through the point $F=g=1$.

The slope Z of this curve is found, as in the case of spherical symmetry, from (5.2) where $a=(\gamma-1)$ and $b=(3\gamma-1)$.

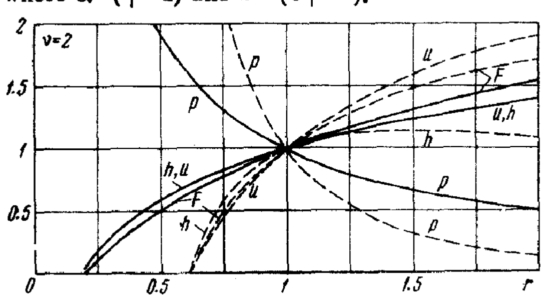

Fig. 5.

Fig. 5 shows the parameter distributions along the nozzle axis for $\varkappa=\text{const}$ obtained by numerical integration of Eq. (6.1), i.e. for $\nu=2$, and system (2.5) for $\nu=3$ (broken curves).

Quite naturally, the functions describing motion with heating must be replaced by relations (3.1) at some distance r where the nozzle walls terminate.

We are now confronted by the question of the uniqueness of the steady-state motion. On the one hand, subsonic motion is impossible. On the other hand, passage through the speed of sound is possible only at the singular point $F=g=h=1$.

As we see from Fig. 4, the singular point is a saddle point through which the unique integral curve passes. The same applies in the spherical case ($\nu=3$) and for $\varkappa \neq$ const, since the behavior of the integral curves is similar to their behavior for Eq. (6.1). According to (5.2), for $\nu=2$ the slope $Z=0.581$ (for $\gamma=5/3$), i.e. even the slope of the integral curves is close to the slope for $\nu=3$.

Of course, in these cases ($\nu=3$ and $\varkappa \neq$ const) one cannot use the results obtained by analyzing (6.1) for large F and g. The line $dg/dF=0$ is described by Eq.

$$F = (\gamma - 1)g(\gamma + 1)^{-1} \;[1-2\varkappa S(\gamma+1)^{-1/2} \;g^{-1/2}]^{-1} \tag{6.3}$$

Thus, for $S \to \infty$ and $\varkappa=$ const, and especially for $\varkappa \to 0$, we find that $F \to g(\gamma-1)/(\gamma+1)$. This means that when the solution goes beyond this limiting straight line ($g \to g_{max}$, $F \to F_{max}$) the heating is immaterial (the thermal energy is much lower than the kinetic energy: $h \ll g$). Hence, we can make use of the results of the above analysis for the case of an adiabatic jet and show that the optical thickness of the hypersonic portion of the jet is finite, so that F^{∞} is also finite. Thus, the steady-state transonic motions with heating are, in fact, unique.

7. Let us consider the dependences of the parameters h_*, p_*, ρ_* in the critical cross section and the mass burnup rate $M^{\cdot}$ on the radiation flux F_* in this cross section and on the radius r_* of the cross section. Since $\varkappa = 1/l\rho$, in the critical cross section we have

$$\lambda r_* = l_* = (\varkappa_* \rho_*)^{-1} \tag{7.1}$$

Assuming that the relationship of $\varkappa$, h, and ρ can be described exponentially (2.7), we obtain

$$r_* = (\varkappa_* \rho_*)^{-1} = (\lambda K)^{-1} h_*^{\alpha} \rho_*^{\beta} \tag{7.2}$$

From (7.2) and (1.3) we have the relations

$$\rho_* = h_*^{\alpha/(\beta+1)} (\lambda K r_*)^{-1/(\beta+1)}$$

$$F_* = \zeta(\nu)(\gamma-1)^{1/2} h_*^{3/2+\alpha/(\beta+1)} [1/2(\gamma+1)+\chi] r_*^{\xi} (\lambda K)^{-1/(\beta+1)}$$

$$M^{\cdot} = \xi(\nu)(\gamma-1)^{1/2} h_*^{1/2+\alpha/(\beta+1)} (\lambda K)^{-1/(\beta+1)} r_*^{\xi}$$

$$p_* = \gamma^{-1} h_*^{1+\alpha/(\beta+1)} (\gamma-1)(\lambda K r_*)^{-1/(\beta+1)}, \qquad \xi = \nu - 1 - 1/(\beta+1) \tag{7.3}$$

Let $\varkappa=$ const. We then have

$$\rho_* = (\lambda \varkappa r_*)^{-1}, \qquad p_* = \gamma^{-1} h_* (\gamma-1)(\lambda\varkappa r_*)^{-1}$$

$$F_* = \zeta(\nu)(\gamma-1)^{1/2} h_*^{3/2} [1/2(\gamma+1)+\chi] r_*^{\nu-2} (\lambda\varkappa)^{-1} \tag{7.4}$$

$$M^{\cdot} = \zeta(\nu)(\gamma-1)^{1/2} h_*^{1/2} r_*^{\nu-2} (\lambda\varkappa)^{-1}$$

With $h_* \gg Q$, when we can set $\chi=0$, and for $\nu=3$ we obtain

$$h_* \sim (F_*\varkappa)^{2/3} r_*^{-2/3} \sim (q_* \varkappa r_*)^{2/3}, \qquad \rho_* \sim (\varkappa r_*)^{-1}$$

$$p_* \sim F_*^{2/3} r_*^{-5/3} \varkappa^{-1/3} \sim q_*^{2/3} (\varkappa r_*)^{-1/3} \tag{7.5}$$

Here we have introduced the radiation flux per unit area of the critical surface, $q_* = F_*/4\pi r_*^2$.

Since the maximum value of the dimensionless enthalpy h_m for each value of γ and ν (with $\chi = 0$) is a definite number found through numerical integration of (2.6) from the point (1.7), the dimensional maximum enthalpy h_{max} is proportional to h_*. In precisely the same way, the quantities p_0, r_0, F_∞ are proportional to p_*, r_*, F_*, respectively. Hence, (7.5) implies that with a given flux q_* the quantity h_{max} increases with increasing body size r_0, while p_0 diminishes. With a fixed total radiation flux F_∞ the quantity p_0 increases with decreasing radius r_0, but somewhat more slowly than $1/r_0^2$. The quantity h_{max} also changes with changing sphere area, but more slowly than this area. The dependence of the pressure p_0 on the absorption coefficient is likewise rather weak (as $1/\varkappa^{1/5}$, rather than as $1/\varkappa^{1/2}$, as in the case of nonsteady-state planar expansion [1]).

We note that the dependence of p_0 on q, r, and $\varkappa$ coincides to within a numerical factor with the dependence [8] on these parameters of the pressure at the center of an expanding layer of constant mass at the instant when the rarefaction wave passes from the edges of this layer to its center (whereupon the entire gas layer begins to expand two- or three-dimensionally; prior to this instant the expansion in the central zone is one-dimensional (planar) [1]).

The author of [8] has already noted that the two-dimensional character of the gas motion becomes significant after this instant.

In fact, with planar motion and a constant mass absorption coefficient, radiation is absorbed in a layer of constant mass, since despite the expansion of the layer, i.e. the increase in its thickness x, the radiation free path $l \sim 1/\rho$ and $\rho \sim m_1/x$, where m_1 is the mass of the layer per unit area, so that the optical thickness $\tau_1 \approx l^{-1}x$ of the one-dimensional zone is constant. Two- or three-dimensional motion begins at the instant when the layer has expanded to a thickness x comparable with its lateral dimension r_0. The radiation-heated layer expands at a rate on the order of the average speed of sound [1] corresponding to the heating of the layer achieved by that instant; the instant is roughly coincident with the instant of meeting in the center of the layer of the rarefaction waves arriving from its lateral edges. From this instant on the pressure gradients level off, as do the characteristic velocities of motion in the lateral and principal directions. Thus, the layer width $r \sim x \tan\theta$, where θ is the average apex angle of the jet and $\tan\theta$ is on the order of unity, so that

$$\rho \approx m_1 r_0 / x^2, \quad d\tau_2 \approx \varkappa dm, \quad dm = \rho dx \approx (m_1 r_0 / x^2)\, dx,$$

i.e. the mass m_2 along the beam path and the optical thickness τ_2 of the two-dimensionally spreading layer diminish. The radiation penetrates into the deeper layers. The optical thickness of the entire two-dimensional portion of the jet is constant,

$$\tau_2 = \int_{r_0}^{\infty} \varkappa\rho\, dx \approx \varkappa m_1 \approx \tau_1$$

with τ_2 determined principally by the optical thickness of those layers of the two-dimensional portion of the jet which are adjacent to the boundary of the one- and two-dimensional portions, and the quantity τ_2 is of the order of the optical thickness τ_1 of the one-dimensional portion.

Steady-state burnup apparently begins with the inception of two-dimensional motion The characteristic parameters in the steady state depend on the magnitude of the radiation flux and initial layer width r_0(i e. on the end face area of an irradiated rod or on the diameter of the irradiated surface spot) in the same way as in the exact problem of the motion of a sphere considered in the present paper. In the latter problem this motion is radial from the very beginning (both during the "actuation of a conical nozzle with straight walls", and in the steady state).

Let us make one further note. Although the motion is steady and the parameters depend solely on the energy flux F, if the total applied energy $E = F\tau$ (τ is the time for which the energy is applied) is held constant while τ is varied, the parameters depend on τ. The pressure $p_0 \sim 1/\tau^{2/3}$ and the pressure impulse $J = p_0\tau \sim \tau^{1/3}$, i. e. it increases with the duration τ in contrast to the case of one-dimensional nonsteady-state expansion [1], where $p_0 \sim 1/\tau$ and J is independent of τ.

This conclusion is, of course, valid only up to a certain value of τ, until h_{max} becomes comparable to Q (i. e. as long as $\chi \ll 1$), after which the increase in impulse with τ ceases.

Let $\alpha = 3/2, \beta = 1$. Then

$$\rho_* = h_*^{3/4} (\lambda K r_*)^{-1/2}, \qquad p_* = (\gamma - 1) h_*^{7/4} (\lambda K r_*)^{-1/2} \gamma^{-1}$$

$$M^{\cdot} = \zeta(\nu) (\gamma - 1)^{1/2} h_*^{5/4} r_*^{(\nu - 3/2)} (\lambda K)^{-1/2} \tag{7.6}$$

$$F_* = \zeta(\nu) (\gamma - 1)^{1/2} h_*^{5/4} r_*^{(\nu - 3/2)} [1/2 (\gamma + 1) + \chi] (\lambda K)^{-1/2}$$

In the spherical case ($\nu = 3$) for $\chi \ll 1$ we find that

$$\rho_* \sim F_*^{1/3} K^{-1/3} \, r_*^{-1} \sim q_*^{1/3} (K r_*)^{-1/3}, \quad M^{\cdot} \sim F_*^{5/9} r_*^{2/3} K^{-2/9} \sim q_*^{5/9} r_*^{16/9} K^{-2/9}$$

$$p_* \sim F_*^{7/9} K^{-7/9} r_*^{-5/3} \sim q_*^{7/9} (K r_*)^{-1/9}, \quad h_* \sim F_*^{4/9} K^{-2/9} r_*^{-2/3} q_*^{4/9} (K r_*)^{2/9} \tag{7.7}$$

Thus, decreases in the radius r_0 of the sphere (changes in the radiation focusing) with a fixed total flux F_∞ produce increases in p_* and p_0 and a slower increase in h_{max}. With a fixed q_*, decreases in r_* are accompanied by decreases in h_*. The dependence of the pressure p_0 on r_0 is especially weak. All of the parameters vary very slowly with changes in the coefficient K in Formula (2. 7), i. e. with changes from one material to another or with changes in the radiation wavelength.

As already noted in the introduction, a necessary condition for the establishment of a steady-state is $\rho_* \ll \rho_0$, where ρ_0 is the normal density of the material constituting the solid. From (7. 4), as from (7 6), it follows that this condition is fulfilled for a body of sufficient size. Another condition is a sufficient heating duration τ : it must be larger than the time required for the establishment of the steady-state.

The authors of [2] consider the problem of propagation through a dense substance of a nonsteady-state self-consistent plane rarefaction and heating wave. The speed at which the rarefaction wave propagates is $(dm/dt) \approx \rho c \approx \rho \sqrt{h}$, where ρ is the characteristic density in the wave ($\rho \ll \rho_s \ll \rho_0$), h is the characteristic value of the enthalpy attained as a result of heating ($h \gg Q \geqslant h_s$), is the corresponding speed of sound; m is the mass of material caught up by the wave As the mass of the heated layer increases, heating of the entire gas layer by the radiation requires a decrease in the characteristic absorption coefficient $\varkappa$. If $\varkappa$ varies in accordance with law (2. 16), the condition that the optical layer thickness τ_0 is the order of unity results in the following relationship of the characteristic parameters:

$$K h^{-\alpha} \rho^{\beta} m \approx 1$$

From the balance relationship for the energy applied to the gas and the energy present in it we obtain

$$hm \approx qt, \qquad h^{\alpha+1} \rho^{-\beta} \approx qKt$$

Substituting these relations into the equation for the speed of progression of the front we obtain

$$\frac{dm}{dt} \approx \frac{h^{\alpha/\beta+1/2}}{(mK)^{1/\beta}} \approx \frac{qt^{\alpha/\beta+1/2}}{m^{(\alpha+1)/\beta+1/2}K^{1/\beta}}$$

Integrating, we obtain the law of motion of the front,

$$m^{3/2+(\alpha+1)/\beta}K^{1/\beta} \approx q^{1/2+\alpha/\beta}t^{3/2+\alpha/\beta}$$

For $\alpha = 3/2$ and $\beta = 1$ this relation leads to the following dependences of the parameters on the flux and time:

$$m \approx q^{1/2}K^{-1/4}t^{3/4}, \qquad x \approx q^{1/4}K^{1/8}t^{9/8}$$
$$h \approx q^{1/2}(Kt)^{1/4}, \qquad p \approx q^{3/4}(Kt)^{-1/8},$$
$$\rho \approx q^{1/4}(Kt)^{-3/8}, \qquad (dm/dt) \approx q^{1/2}(Kt)^{-1/4}$$

Here p is the characteristic pressure and $x = m/\rho$ is the characteristic thickness of the self-consistent wave. The effects of nonunidimensional motion make themselves felt at the instant when $x \approx r_0$, where r_0 is the dimension of the irradiated area Hence, the instant of inception of two- or three-dimensional motion and the parameters at this instant can be found from the following expressions:

$$t \approx r_0 h^{-1/2} \approx r_0^{8/9} q^{-2/9} K^{-1/9}, \qquad h \approx q^{4/9}(Kr_0)^{2/9}, \qquad p \approx q^{7/9}(Kr_0)^{-1/9}$$
$$\rho \approx q^{3/9}K^{-1/3}r_0^{-1/3}, \qquad M_* \approx (dm/dt)\, r_0^2 \approx q^{5/9}r_0^{16/9}K^{-2/9}$$

Taking $r_0 \approx r_*$, we find that the laws of variation of all the parameters coincide with those cited above for the steady-state We note that although the dependences of h, p, and ρ on the characteristic dimension r_0 for a fixed flux q on the surface of the body are very weak, it is precisely the nonunidimensional character of the motion which leads to the establishment of a steady-state, while the motion in a planar wave is nonsteady

Thus, soon after the instant when the rarefaction waves penetrate from the side surfaces of the jet to its center the temperature stops increasing, the density stops decreasing, and the rate of burnup dm/dt stops diminishing

If the motion is not strictly radial (if, for example, the parallel beam of rays falls on the end face of a rod or on some limited area of a plane), then the picture of motion and such parameters as the mass burnup rate and maximum temperature can, of course, differ from those obtained in the problem of radial motion of a gas heated by a radial radiation flux, just as the mass expenditures in nozzles of various shapes differ from each other for the same critical cross section Nevertneless, for estimates of more complex motions we can make use of results obtained through precise computation of the problem on the radial motion and heating, and especially through the similarity laws for steady-state burnup derived above Here we must assume that the shape of the body does not change during burnup (these similarity laws can also be obtained by simple dimensional analysis)

8. Let us consider some numerical examples Paper [5] contains a discussion of the problem of heating of ionized gases by laser radiation up to high temperatures on the order of 10^{10} K. At such temperatures the internal energy e of a unit mass of deuterium plasma is 1.3×10^{15} erg/g, so that the enthalpy $h = \gamma e \approx 2 \times 10^{15}$ erg/g. For a density $\rho = 10^{-2}$ g/cm^3 the free path ℓ of radiation of wavelength 6000 Å is approximately 10^{-2} cm, so that the absorption coefficient $\varkappa \approx 10^4$ cm^2/g Hence, $K = \varkappa h^{3/2}\rho^{-1} \approx 10^{29}$ CGS units. The value $h \approx 2 \times 10^{15}$ erg/g for $r_* \approx 10^{-2}$ cm can be attained if $F_* \approx 10^{17}$ erg/sec $= 10^{10}$ W $= 10$ J/nsec (i. e. $q_* \approx 10^{20}$ erg/cm^{2}sec $= 10^{13}$ W/cm^2 $= 10^4$ J/cm^{2}nsec)

If the heat conduction loss is neglected, then, when the radiation is confined to a cone rather than falling from all directions, the true applied flux is essentially $F_* \Omega / 4\pi$, where Ω is the solid angle of the cone. One should bear in mind, however, that with a narrow cone or flat channel, the wall length can become an important parameter: further, it may turn out that heat transfer through the walls is not negligible. Electronic heat conduction plays a substantial role in the example being considered.

In fact, since the density of the radiation-heated gas must be smaller than 10^{-2} g/cm^3 (the electron concentration $n < 3 \times 10^{21}$ 1/cm^3 and no radiation is reflected [5]), it follows that the condition $r_* \gtrsim 10^{-2}$ cm must be fulfilled for the given value of F_*.

In order to guarantee sufficient time for the steady-state to be established it is necessary that the material layer expand to distances larger than r_* : hence, the time required for establishment of the steady-state is of order r_*/u_*. This is the condition of the required duration of energy application (an energy application process lengthy from the gas dynamic standpoint is often very brief on the ordinary time scale). With an expansion velocity u_* of 5×10^7 cm/sec the steady-state establishment time is of order 0.2 nsec.

At such high temperatures the effect of electronic heat conduction becomes considerable even with times of order of nanoseconds For $T \approx 10^{7\,\circ}K$ we find the $k_e T = 4 \times 10^{18}$ erg/cm sec, where k_e is the coefficient of heat conduction $(k_e \sim T^{5/2})$ and $\rho_* C_V T = 10^{13}$ erg/cm^3 (for $\rho_* = 10^{-2}$ g/cm^3 where C_V is the specific heat per unit mass. Hence, the coefficient of thermal diffusivity $a = k_e / \rho C_V \approx 4 \times 10^2$ cm^2/sec. Heat propagates to a distance of order $\sqrt{at}$, i.e. to a distance of order 2×10^{-2} cm for $t = 10^{-9}$ sec. Hence, the motion is somewhat complicated by the influence of heat conduction in the example under consideration. For a smaller total flux F_* the quantity h_*, and therefore the temperature T_*, would be smaller, making possible the use of the equations and solutions in which heat conduction is not taken into account. Thus, for $F_* \approx 10^7$ W (i.e. for $q_* \approx 10^{10}$ W/cm^2) the quantity h_* is equal to about 10^{14} erg/g $(T_* \approx 5 \times 10^{5\,\circ}$ K), $u_* \approx 10^7$ cm/sec = 100 km/sec, and with $r_* \approx 10^{-2}$ cm the time of establishment of the steady-state is of order 10^{-9} sec. The density ρ_* is approximately 10^{-3} g/cm^3, and the burnup rate with a sphere density of 1 g/cm^3 is just 10^4 cm/sec, i.e. the boundary of the sphere shifts by 10^{-3} cm in 10^{-7} sec, so that the radius of the sphere remains practically unchanged. The pressure p_* in our example is very high, of order 10^4 kg/cm^2.

Let us make some notes concerning the applicability of the above formulas. In the course of our discussion we assumed that for any T and ρ the absorption coefficient varies in accordance with the law $\varkappa \sim T^{-3/2} \rho$, i.e. that it increases with decreasing temperature. But for $T \lesssim 10^{4\circ}$K, when the gas is not ionized, the absorption coefficient can drop sharply if the radiation wavelength lies in the optical range. There then arises the problem of heating of the material from a temperature of order $10^{4\circ} K$, i.e. to the plasma state. This problem does not arise if the radiation is so powerful that the energy flux at the sublimating surface

$$q_s = F_\infty / 4\pi r_0^2$$

(the vapor does not absorb prior to its breakdown) is sufficient to produce breakdown (according to the data of [6] and [9], the induction of breakdown by radiation of energy $h\nu \approx 2$ eV in a cold gas with an ionization potential $I \approx 15$ eV requires a flux, $q_1 = 10^{11}$ W/cm^2 or $F_1 \approx 10^8$ W for $r^0 = 10^{-2}$ cm in the case where $\rho \approx 10^{-3}$ g/cm^3; q_1 diminishes with increasing pressure p [6 and 9] to values of order $1\text{-}10^9$ W/cm^2). Vapor is released at a temperature of order of I/k or even higher (with superheating in the liquid phase), so that the breakdown conditions change. With a sufficiently low ionization potential of the vapor material or of impurities present, the equilibrium concentration of electrons and the resultant absorption through collisions with neutral atoms or ions [6] can make itself felt. This results in self-heating of the material. It is clear that with increasing T the quantity q_1 must also diminish due to the appearance of excited levels from which electrons can be "plucked off" by long-wave radiation by way of the photoelectric effect. Nor does the above problem arise in the case where the radiation contains a sufficient number of quanta of energy $h\nu > I$, i. e. quanta capable of producing ionization which are absorbed at small depths ℓ in the cold gas (usually [6] $\ell \approx 10^{-2}$ cm for $\rho \approx 10^{-3}$ g/cm^3). These quanta can be either present in the incident flux or arise in the heated gas itself.

We shall not consider this physical problem, and bring to a close our discussion of the hydrodynamic problem of selecting a constant material burnup rate and determining the steady-state with lateral spreading flow of the radiation-heated vapor jet.

BIBLIOGRAPHY

1. Nemchinov, I. V., Razlet ploskogo sloia gaza pri postepennom vydelenii energii (Expansion of a plane gas layer with gradual liberation of energy). PMTF No. 1, pp. 17-26, 1961.
2. Afanas'ev, Iu. V., Krol', V. M., Krokhin, O. N. and Nemchinov, I. V., Gazodinamicheskie protesessy pri nagrevanii veshchestva izlucheniem lazera (Gas dynamic processes in heating of a substance by laser radiation). PMM Vol. 30, No. 6, 1966.
3. Howarth, L. (ed.), Sovremennoe sostoianie aerodinamiki bol'shikh skorostei (Modern High-Velocity Aerodynamics), Vol. 1, Chpt. 5, Sec. 3, Radial'noe techenie (Radial Flow), pp. 163-166); Chpt. 6, Odnomernoe techenie (One-dimensional Flow), pp. 194-222; Chpt. 3, Sec. 12, Neustanovivsheesia techenie (Unsteady Flow), pp. 101-103, Izd. - inostr. lit., Moscow, 1955.
4. Courant, R. and Friedrichs, K., Sverkhzvukovoe techenie i udarnye volny (Hypersonic Flow and Shock Waves), Chpt. 5, Techenie v soplakh i struiakh (Nozzle and jet flow), Sec. 145, Techenie v konuse (Flow in a Cone), pp. 349-351, Izd. inostr. lit., Moscow, 1950.
5. Basov, N. G. and Krokhin, O. N., Usloviia razogreva plazmy izlucheniem opticheskogo kvantovogo generatora (Conditions of plasma heating with laser radiation). ZhETF Vol. 46, No. 1, 1964.
6. Zel'dovich, Ia. B and Raizer, Iu. P., Fizika udarnykh voln i vysokotemperaturnykh gidrodinamicheskikh iavlenii (Physics of Shock Waves and High-temperature Hydrodynamic Phenomena), Chpt. 5, Pogloshchenie i ispuskanie izlucheniia v gazakh pri vysokikh temperaturakh (Absorption and Emission of Radiation in Gases at High Temperatures), Part 5, Proboi i nagrevanie gaza pod deistviem sfokusirovannogo lazernogo lucha (Breakdown and Heating of Gas under the Action of a Focused Laser Beam), Sec. 22, pp. 289-293, Izd. "Nauka", Moscow, 1966.
7. Sauer, R., Techeniia szhimaemoi zhidkosti (Compressible Fluid Flow). Chpt. 1, Sec. 5, Osnovy teorii neustanovivshegosia techeniia v trubakh (Basic Theory of Unsteady Flow in Pipes), pp. 26-28, Izd. inostr. lit., Moscow, 1954.

8. Nemchinov, I. V., Razlet podogrevaemoi massy gaza v reguliarnom rezhime (Expansion of a heated gas mass under regular conditions). PMTF No. 5, 1964.
9. Raizer, Iu. P., Proboi i nagrevanie gazov pod deistviem lazernogo lucha (Breakdown and heating of gases under the action of a laser beam). Uspekhi fiz. Nauk Vol. 87, No. 1, 1965.

Translated by A.Y.

A STEADY-STATE PROBLEM OF HEAT CONDUCTION IN A LAYER WITH HEAT TRANSFER CONDITIONS AT THE BOUNDARY

(STATSIONARNAIA ZADACHA TEPLOPROVODNOSTI V SLOE S USLOVIIAMI TEPLOOTDACHI NA GRANITSE)

PMM Vol. 31, No. 2, 1967, pp. 320-327

V. N. PED' and I. B. SIMONENKO
(Rostov-on-Don)

(Received March 30, 1966)

The following problem was investigated. A layer $(-a \leqslant z \leqslant a)$ of thickness $2a$ releases heat into the surrounding space in accordance with Newton's law,

$$\lambda \partial u / \partial n + ku = 0, \qquad z = \pm a \tag{0.1}$$

Here $\lambda (\geq 0)$ is the coefficient of heat conduction; $k (< 0)$ is the coefficient of heat transfer: the temperature of space surrounding the layer is assumed equal to zero; $\partial / \partial n$ is differentiation with respect to the exterior normal.

In the midplane of the layer $(z = 0)$ lies a disk of unit radius with its center at the point $(0, 0, 0)$. The disk is assumed to be at the temperature

$$u|_D = g \tag{0.2}$$

It is also assumed that the function $g \in C_2$ (i.e. that it is doubly continuously differentiable). We are required to find the steady-state thermal field u in the layer without sources, i.e. the function u at all internal points of the layer (except at points on the disk) which satisfies the Laplace condition and the condition at infinity

$$\Delta u = 0; \quad u(x, y, z) \Rightarrow 0, \text{ for } \quad (x, y, z) \to \infty \tag{0.3}$$

The symbol $\Rightarrow$ denotes uniform convergence. In the present paper we shall find the asymptotic form of the solution for $k \to 0$ and $k \to \infty$.

The most curious case is that of the asymptotic form for $k \to 0$ (Sec. 6), which cannot be arrived at formally, and requires "nonformal" investigation of the influence function. When the layer is replaced by a bounded body, this asymptotic form can be obtained formally and can be written as

$$k^{-1} u_{-1} + u_0 + k u_1 + k^2 u_2 + \ldots \tag{0.4}$$